Subtracting Mixed Numbers

Introduction:

"Now that you know how to add and subtract fractions and add mixed numbers, let's investigate two of the many methods to subtract mixed numbers. Keep in mind that mixed numbers include whole numbers and fractional parts, so not only do we need to subtract the fractions, but the whole numbers as well, just like when we added mixed numbers. The two methods that we're learning today are **using the number line** and **decomposition**. Let's use the number line first. Our first example will have common denominators."

Note: Have students make two columns on their paper so they can write their examples and you tries side-by-side.

Example: $5\frac{3}{8}-2\frac{7}{8}$

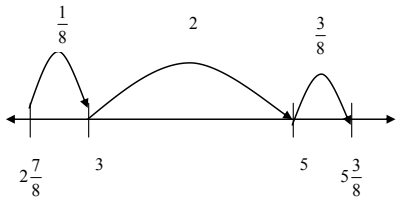
"There is no one way to use the number line. You can make forward jumps, backward jumps, and in any increment that makes sense to you. When we're subtracting $2\frac{7}{8}$ from $5\frac{3}{8}$, we're basically finding the distance between the two numbers on the number line."

"My first jump will be to the next whole number which is 3. That is $\frac{1}{8}$ on the number line which I've notated above. Then I'm going to make a jump of 2 which moves me to 5 on the number line. The last jump will be to $5\frac{3}{8}$, which is $\frac{3}{8}$ more on the number line. Now we add all of our jumps together. So we need to add $\frac{1}{8}+2+\frac{3}{8}$."

Write:

Example #1

$$5\frac{3}{8}-2\frac{7}{8}$$



$$=\frac{1}{8}+2+\frac{3}{8}$$
$$=2+\frac{1}{8}+\frac{3}{8}$$
$$=2+\frac{1+3}{8}$$
$$=2+\frac{4}{8}$$
$$=2\frac{4}{8}$$
$$=2\frac{1}{2}$$

"Our fractions already have common denominators so we can add our fractions together, along with our wholes. So $2+\frac{1}{8}+\frac{3}{8}=2\frac{4}{8}$. Thumbs up, if you agree."

[There should be some thumbs down because the answer is not in simplest form.]

"Turn to your table partner and tell them why we aren't finished. Who can tell me what I need to do?"

[Put the answer in simplest form.]

"What is $2\frac{4}{8}$ in simplest form?" $[2\frac{1}{2}]$

"Now let's do that same problem using decomposition. First, I'm going to break apart the mixed numbers into a sum of its wholes and fractional parts. Because I am subtracting the entire quantity of $2+\frac{7}{8}$, I must distribute the negative or take the opposite of both terms inside the parentheses. So the opposite of $+2$ is -2, and the opposite of $+\frac{7}{8}$ is $-\frac{7}{8}$. Now I will rearrange my numbers so the whole numbers are together, as well as the fractions. I can subtract my wholes, but I can't subtract my fractions without ending up with a negative fraction. So I'll decompose 3 into $2+1$ and find an equivalent fraction for 1, which is $\frac{8}{8}$. Now we have $\frac{8}{8}+\frac{3}{8}$ which equals $\frac{11}{8}$."

"Can we subtract the fractions now?" [yes]

Write:

Example #1

$$5\frac{3}{8}-2\frac{7}{8}$$

$$=\left(5+\frac{3}{8}\right)-\left(2+\frac{7}{8}\right)$$
$$=5+\frac{3}{8}-2-\frac{7}{8}$$
$$=5-2+\frac{3}{8}-\frac{7}{8}$$
$$=3+\frac{3}{8}-\frac{7}{8}$$
$$=2+1+\frac{3}{8}-\frac{7}{8}$$
$$=2+\frac{8}{8}+\frac{3}{8}-\frac{7}{8}$$
$$=2+\frac{11}{8}-\frac{7}{8}$$
$$=2+\frac{4}{8}$$
$$=2\frac{4}{8}$$
$$=2\frac{1}{2}$$

"What is $\frac{11}{8}-\frac{7}{8}$?" $[\frac{4}{8}]$ "What is $\frac{4}{8}$ in simplest form?" $[\frac{1}{2}]$

"We arrive at the same answer of $2\frac{1}{2}$. One of the great things about decomposition is that everyone does not have to decompose the same way, but we can all arrive at the correct answer!"

"Let's try another example with unlike denominators."

Example : $2\frac{1}{3}-1\frac{1}{2}$

"Again, on the number line, we are calculating the distance between $1\frac{1}{2}$ and $2\frac{1}{3}$. As I mentioned before, you can make backward jumps as well. So I'm going to move backward on this example. My first jump will be from $2\frac{1}{3}$ to 2. That is a jump of $\frac{1}{3}$. Then I'm going to jump from 2 to $1\frac{1}{2}$, which is a movement of $\frac{1}{2}$. Now I simply need to add $\frac{1}{3}$ and $\frac{1}{2}$."

"What is the LCM of 3 and 2?" [6]

"What do I need to multiply $\frac{1}{3}$ by to get an equivalent fraction with a denominator of 6?" [$\frac{2}{2}$]

"What do I need to multiply $\frac{1}{2}$ by to get an equivalent fraction with a equivalent fraction with a denominator of 6?" [$\frac{3}{3}$]

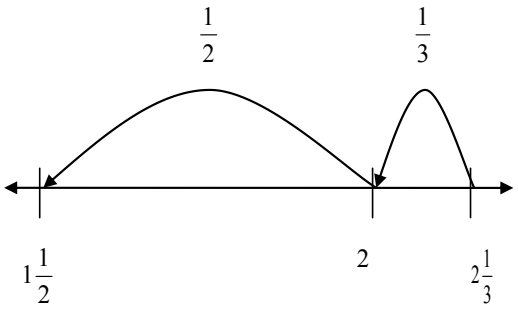
"Now I can add my fractions. $\frac{2}{6}+\frac{3}{6}$ equals?" [$\frac{5}{6}$]

"Is $\frac{5}{6}$ in simplest form?" [yes]

Write:

Example #2

$2\frac{1}{3}-1\frac{1}{2}$



$$=\frac{1}{3}+\frac{1}{2}$$
$$=\frac{1}{3}\left(\frac{2}{2}\right)+\frac{1}{2}\left(\frac{3}{3}\right)$$
$$=\frac{2}{6}+\frac{3}{6}$$
$$=\frac{2+3}{6}$$
$$=\frac{5}{6}$$

"How do you know? Turn to your table partner to discuss." [because they have no common factors besides 1.]

"Now let's try the same example using decomposition. You should write it next to the number line example we just did. Again, I'm going to break apart the mixed numbers into a sum of its parts. I'm subtracting the whole quantity of 1 and $\frac{1}{2}$."

"So what do I need to do next?" [distribute the negative or take the opposite of both terms in the parentheses]

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"I rearrange my terms so I have my wholes together and my fractions together I can subtract my wholes, and I end up with 1. This time I'm going to decompose differently than the first example. Since I need to subtract $\frac{1}{2}$, I'm going to decompose 1 into $\frac{1}{2}+\frac{1}{2}$. Now we have $+\frac{1}{2}-\frac{1}{2}$ which equals 0 so I can cross it out. Now we're left with $\frac{1}{2}+\frac{1}{3}$. Again, I need to make equivalent fractions of $\frac{1}{2}$ and $\frac{1}{3}$ with a new denominator of 6."

"What should I multiply $\frac{1}{2}$ by?" $[\frac{3}{3}]$

"What should I multiply $\frac{1}{3}$ by?" $[\frac{2}{2}]$

Write:

Example #2

$$2\frac{1}{3}-1\frac{1}{2}$$

$$=\left(2+\frac{1}{3}\right)-\left(1+\frac{1}{2}\right)$$
$$=2+\frac{1}{3}-1-\frac{1}{2}$$
$$=2-1+\frac{1}{3}-\frac{1}{2}$$
$$=1+\frac{1}{3}-\frac{1}{2}$$
$$=\frac{1}{2}+\cancel{\frac{1}{2}}+\frac{1}{3}-\cancel{\frac{1}{2}}$$
$$=\frac{1}{2}+\frac{1}{3}$$
$$=\frac{1}{2}\left(\frac{3}{3}\right)+\frac{1}{3}\left(\frac{2}{2}\right)$$
$$=\frac{3}{6}+\frac{2}{6}$$
$$=\frac{3+2}{6}$$
$$=\frac{5}{6}$$

"Now that there is a common denominator, the fractions can be added. The sum of $\frac{3}{6}+\frac{2}{6}$ is?" $[\frac{5}{6}]$

"You are all going to do the next example with your table partner. You will work through the problem together, but each of you must write it in your own notebook. Please do both strategies side-by-side. Once you're finished, you'll show it to me and get a piece of chart paper to chart your work. Once everyone is done, we will do a gallery walk to see how each pair solved the problem."

"Is there anyone who doesn't understand the directions?" [no (hopefully!)]

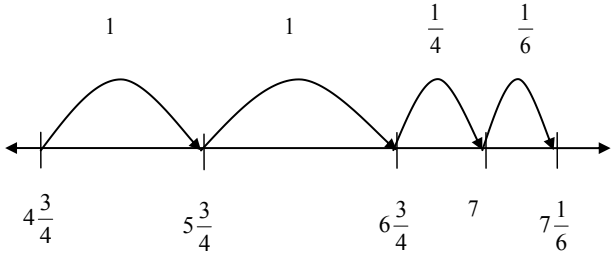
Here is the problem: $7\frac{1}{6}-4\frac{3}{4}$

As time allows, have students who solved it differently explain their thinking to the class.

The teacher can also put up a chart once students finish. Try to decompose it differently than the students. Here is an example.

You Try: $7\frac{1}{6}-4\frac{3}{4}$

Number Line



$$=1+1+\frac{1}{4}+\frac{1}{6}$$
$$=2+\frac{1}{4}+\frac{1}{6}$$
$$=2+\frac{1}{4}\left(\frac{3}{3}\right)+\frac{1}{6}\left(\frac{2}{2}\right)$$
$$=2+\frac{3}{12}+\frac{2}{12}$$
$$=2+\frac{3+2}{12}$$
$$=2+\frac{5}{12}$$
$$=2\frac{5}{12}$$

Decomposition

$$7\frac{1}{6}-4\frac{3}{4}$$
$$=\left(7+\frac{1}{6}\right)-\left(4+\frac{3}{4}\right)$$
$$=7+\frac{1}{6}-4-\frac{3}{4}$$
$$=7-4+\frac{1}{6}-\frac{3}{4}$$
$$=3+\frac{1}{6}-\frac{3}{4}$$
$$=2+1+\frac{1}{6}-\frac{3}{4}$$
$$=2+\frac{1}{4}+\cancel{\frac{3}{4}}+\frac{1}{6}-\cancel{\frac{3}{4}}$$
$$=2+\frac{1}{4}+\frac{1}{6}$$
$$=2+\frac{1}{4}\left(\frac{3}{3}\right)+\frac{1}{6}\left(\frac{2}{2}\right)$$
$$=2+\frac{3}{12}+\frac{2}{12}$$
$$=2+\frac{3+2}{12}$$
$$=2+\frac{5}{12}$$
$$=2\frac{5}{12}$$

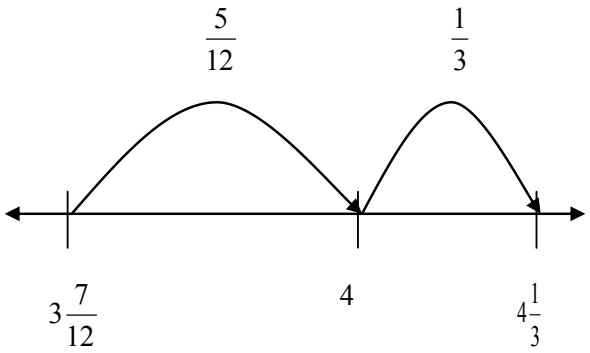
Here is an additional you try that can be used as an exit card. You can also use it within the lesson, if necessary. It is to be done independently so you can see if the students have grasped the concept.

You Try: $4\frac{1}{3}-3\frac{7}{12}$

Possible Solutions

Number Line

$$4\frac{1}{3}-3\frac{7}{12}$$



$$=\frac{5}{12}+\frac{1}{3}$$
$$=\frac{5}{12}+\frac{1}{3}\left(\frac{4}{4}\right)$$
$$=\frac{5}{12}+\frac{4}{12}$$
$$=\frac{5+4}{12}$$
$$=\frac{9}{12}$$
$$=\frac{3\cdot 3}{3\cdot 4}$$
$$=\frac{3}{4}$$

Decomposition

$$4\frac{1}{3}-3\frac{7}{12}$$

$$=\left(4+\frac{1}{3}\right)-\left(3+\frac{7}{12}\right)$$
$$=4+\frac{1}{3}-3-\frac{7}{12}$$
$$=4-3+\frac{1}{3}-\frac{7}{12}$$
$$=1+\frac{1}{3}-\frac{7}{12}$$
$$=\frac{5}{12}+\frac{7}{12}+\frac{1}{3}-\frac{7}{12}$$
$$=\frac{5}{12}+\frac{1}{3}$$
$$=\frac{5}{12}+\frac{1}{3}\left(\frac{4}{4}\right)$$
$$=\frac{5}{12}+\frac{4}{12}$$
$$=\frac{5+4}{12}$$
$$=\frac{9}{12}$$
$$=\frac{3\cdot 3}{3\cdot 4}$$
$$=\frac{3}{4}$$